The relations we have derived enable us to determine not only the effective transmissibility of a medium with circular inclusions of arbitrary concentration, but also to obtain an analytic solution of the two-dimensional problem of filtration in a medium with translational symmetry of inclusions. The analytic solution can be obtained with any accuracy. The solution we have presented was obtained in the third approximation with an accuracy of $\sim \varepsilon^8$. The method used can also be applied to solve transmissibility problems in a medium with inclusions of arbitrary shape. It is very interesting that when the concentration of the inclusions is not low, the quantity u_0, characterizing the flux density at infinity for a problem with widely spaced inclusions, loses its original physical meaning, and the solution obtained corresponds to filtration of an average flux density different from u_0. We note that a similar effect occurs also in treating transport processes in a crystal lattice. This fact must also be taken into account in treating elasticity problems, for whose solution the method used in the present article was originally developed.

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THERMAL BOUNDARY LAYER ON A CYLINDRICAL GAS COLUMN WITH DISTRIBUTED HEAT SOURCES

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UDC 553.6.011

The study of stream interaction with a gas domain where energy liberation occurs is of practical and theoretical interest. We speak of problems when the leaking gas passes through the heat liberation space. The situation mentioned can occur in meterology, in stream heating in an electric arc or other form of electrical discharge, in the air cooling of stabilized gas heat-liberating elements in reactors, in powerful electron beam or other kinds of penetrating radiation propagation in a gas medium, etc. However, systematic computations of the flow and heat-transfer patterns have been executed in application to conditions for longitudinally air-cooled stabilized arcs. Their results are shown most completely in [1-4]. Semiempirical numerical [2, 4] and integral [1, 3] methods were used. There is also a number of theoretical papers of general nature on flows with distributed heat supply (see [5] and the citations there) and a cycle of investigations devoted to laser beam propagation and discharges on a substance (see [6]) which are primarily of estimating nature in the theoretical part.

The purpose of this paper is to compute the thermal boundary layer being formed during air cooling of a cylindrical gas column with arbitrary volume heat sources by an unbounded stream. The stationary problem is examined under the assumption that the main heat elimination mechanism is heat conduction. We limit ourselves to the case of longitudinal blowing around the column of heat liberating gas. In the reference system coupled to the free stream the problem is formulated differently: determine the perturbation of the gas state by the moving distributed heat sources.

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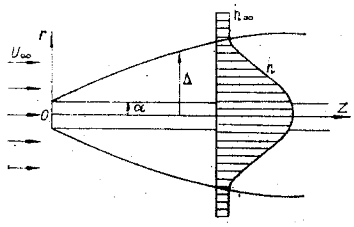

Fig. 1

1. We introduce a cylindrical coordinate system with z axis directed along the stream and origin at the center of the initial section of a gas column of radius a around which the stream is blown (Fig. 1). The flow parameters are constants in the free stream. In the case of an unbounded stream and no gas vortex, the problem can be simplified since the momentum equation allows the integral [1, 2]

$$u = u_\infty = \text{const},\; p = \text{const}, \tag{1.1}$$

where u is the axial velocity and p is the pressure. The remaining continuity and energy equations can be solved by using the apparatus of boundary layer theory:

$$\frac{\partial}{\partial r}(r\rho v) + r u_\infty \frac{\partial \rho}{\partial z} = 0; \tag{1.2}$$

$$\rho v \frac{\partial h}{\partial r} + \rho u_\infty \frac{\partial h}{\partial z} = q + \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\lambda}{c_p}\frac{\partial h}{\partial r}\right); \tag{1.3}$$

$$\rho h = \rho_\infty h_\infty. \tag{1.4}$$

Here v is the radial velocity; h, specific enthalpy; ρ, density; λ, coefficient of heat conduction; and c_p, specific heat of the gas. The source density q differs from zero for $r \le a$ and can be represented in the general case in the form

$$q = b_\alpha \rho^\alpha,\; b_\alpha = \begin{cases} b_\alpha = \text{const} & (r \leqslant a), \\ 0 & (r > a), \end{cases} \tag{1.5}$$

where b_α is a factor dependent on secondary parameters for gasdynamics. For $\alpha = 1$ we have the energy liberation law due to ionization energy losses, say, during the passage of charged particles through a substance [7]. In other words, this case corresponds to the model of a stabilized electron beam of radius a blown off by a gas. If the expression for the Joulean heat is taken instead of (1.5) and Ohm's law is appended to (1.2)-(1.4), then we arrive at the equations of an arc [1-4]. The system of equations (1.2) and (1.3) should be supplemented by the boundary conditions

$$\begin{gathered} v|_{r=0} = 0,\; h|_{z=0} = h_\infty,\; h|_{r\to\infty} = h_\infty, \\ \partial h/\partial r|_{r=0} = 0,\; v|_{z=0} = 0,\; \partial h/\partial r|_{r\to\infty} = 0. \end{gathered} \tag{1.6}$$

2. To solve (1.2)-(1.4), we, following [1, 3], apply an approximate method analogous to the Karman–Pohlhausen integral method. We define the finite thermal layer thickness $\Delta(z)$ as the magnitude of the domain where the thermal action on the stream is substantial (see Fig. 1). On the boundary $r = \Delta$ of the thermal layer the gas enthalpy differs from the unperturbed stream enthalpy h_∞ by a certain small quantity. Integrating (1.2) and (1.3) with respect to r within the thermal layer limits, taking account of (1.6), and eliminating $v(\Delta, z)$, we obtain

$$\frac{d}{dz}\Delta_h^2 = \frac{b_\alpha \rho_m^\alpha a^2}{u_\infty \rho_\infty h_\infty}, \tag{2.1}$$

where

$$\Delta_h^2 = \int_0^\Delta 2r\frac{\rho}{\rho_\infty}\left(\frac{h}{h_\infty} - 1\right)dr = \Delta^2 \int_0^1 2\eta\left(1 - \frac{\rho}{\rho_\infty}\right)d\eta = k_1\left(1 - \frac{\rho_m}{\rho_\infty}\right)\Delta^2.$$

Here $\eta = r/\Delta$, the subscript m indicates the value of the variables on the axis. The quantity Δ_h represents the thickness of the layer acquiring the enthalpy. The mean gas density within the thermal layer becomes small as z grows, and $\Delta_h \to \Delta$ according to (2.2). To solve the problem, it is sufficient to use a one-parameter approximation of the enthalpy profile within the thermal layer limits

$$h = h_\infty + (h_m - h_\infty)f(\eta) \tag{2.3}$$

with the boundary conditions

$$f(0) = 1,\ f'(0) = 0,\ f(1) = 0,\ f'(1) = 0,\ f''(1) = 0, \ldots \tag{2.4}$$

The polynomial $f(\eta) = (1 - \eta)^N(1 + N\eta)$, where $N = 2$ if smoothing of the enthalpy profile on the layer boundary is performed in the first derivative, $N = 3$ if in the first and second, etc., satisfies the conditions (2.4). Polynomials of third and fourth power are ordinarily utilized in numerical computations. The coefficient k_1 in (2.2) is generally a function of ρ_m/ρ_∞

$$k_1 = \int_0^1 \frac{2\eta f(\eta)\,d\eta}{\frac{\rho_m}{\rho_\infty} + \left(1 - \frac{\rho_m}{\rho_\infty}\right) f(\eta)}. \tag{2.5}$$

According to (2.5), $k_1 \simeq 1$ for $\rho_m \ll \rho_\infty$ and $k_1 < 1$ for $\rho_m \sim \rho_\infty$. It is allowable to set it constant ($k_1 \simeq 1$) because it can be expected that the influence of the inaccuracies admitted in the initial section will vanish with distance from the beginning of the layer. An analogous remark can be made about the factor being obtained in the integration of (1.5), which is set equal to one in the right side of (2.1). Used as the second differential equation is (1.3) on the axis. Setting (2.3) into (1.3) and taking account of (1.6), we find

$$\rho_m u_\infty \frac{dh_m}{dz} = b_\alpha \rho_m^\alpha - \frac{2N(N+1)}{\Delta^2}\frac{\lambda_m}{c_{pm}}(h_m - h_\infty). \tag{2.6}$$

Equations (1.4), (2.1), (2.2), and (2.6) form a complete system to determine ρ_m, h_m, Δ_h, Δ. Introducing the dimensionless variables

$$\psi = \Delta_h^2/a^2,\ g = h_m/h_\infty,\ x = b_\alpha \rho_\infty^{\alpha-1} z/u_\infty h_\infty$$

and setting

$$\lambda_m/c_{pm} = (\lambda_\infty/c_{p\infty})s(g),$$

we reduce the system of equations to the form

$$d\psi/dx = g^{-\alpha},\ dg/dx = g^{1-\alpha} - \Lambda s(g)(g-1)^2/\psi, \tag{2.7}$$

where

$$\Lambda = \frac{2N(N+1)k_1}{a^2}\frac{\lambda_\infty h_\infty}{c_{p\infty} b_\alpha \rho_\infty^\alpha}.$$

The problem therefore contains a single similarity criterion Λ, whose physical meaning is evident from (2.6). The boundary conditions are $g(0) = 1$, $\psi(0) = 0$.

3. The solution of (2.7) near the initial section can be written in the series

$$\psi,\ g = x(1 - (\alpha/2)x + \ldots),\ g = 1 + x + \ldots \tag{3.1}$$

The coefficients for the subsequent terms of the expansion are awkward, depend on Λ, and are not presented. The singularity in the right side of the second equation in (2.7) is therefore apparent for $x = 0$.

The case of small Λ is of greatest interest, when the heat influx from the sources predominates over the heat elimination (the case $\Lambda > 1$ corresponds to the problem of small perturbations of the stream properties by energy sources [5]). For $\Lambda = 0$ the following simple solution of (2.7) is valid:

$$g = (1 + \alpha x)^{1/\alpha},\ \psi = \frac{1}{\alpha}\ln(1 + \alpha x) = \ln g, \tag{3.2}$$
$$f_1 = \frac{\Delta^2}{a^2} = \frac{1}{k_1 \alpha}\frac{(1 + \alpha x)^{1/\alpha}\ln(1 + \alpha x)}{(1 + \alpha x)^{1/\alpha} - 1}.$$

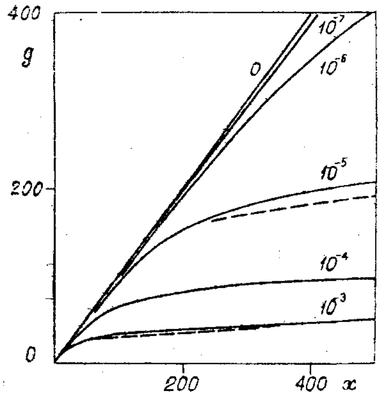

Fig. 2

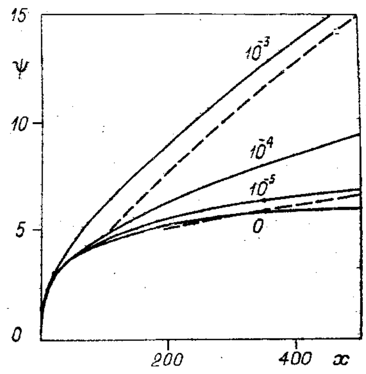

Fig. 3

The solution (3.2) yields the maximal value of the enthalpy (thinning out), which can be achieved on the axis of the thermal layer. The presence of heat elimination will naturally result in smaller quantities. According to (3.2), the thickness of the enthalpy acquisition layer grows considerably more slowly than the enthalpy itself.

It is also easy to find the asymptotic of the solution of the problem for $x \gg 1$. As $x \to \infty$, we have $g \to \infty$, $\psi \to \infty$, $\psi' \to 0$, $g' \to 0$. Setting $s(g) = g^{\beta}$, we find from (2.7)

$$\psi = \Lambda g^{1+\alpha+\beta}. \tag{3.3}$$

It hence follows that

$$\begin{gathered} g(x)^{1+2\alpha+\beta} - g(x_1)^{1+2\alpha+\beta} = \frac{1+2\alpha+\beta}{1+\alpha+\beta}\frac{x-x_1}{\Lambda}, \\ \psi(x)^{\frac{1+2\alpha+\beta}{1+\alpha+\beta}} - \psi(x_2)^{\frac{1+2\alpha+\beta}{1+\alpha+\beta}} = \frac{1+2\alpha+\beta}{1+\alpha+\beta}\Lambda^{\frac{\alpha}{1+\alpha+\beta}}(x-x_2). \end{gathered} \tag{3.4}$$

which are valid for $x > x_1(\Lambda) \gg 1$ and $x > x_2(\Lambda) \gg 1$, respectively. If $x \gg x_1$, $g(x) \gg g(x_1)$ and $x \gg x_2$, $\psi(x) \gg \psi(x_2)$, then the following simple expressions are obtained, respectively, for g and ψ

$$g = \left(\frac{1+2\alpha+\beta}{1+\alpha+\beta}\right)^{\frac{1}{1+2\alpha+\beta}}\left(\frac{x}{\Lambda}\right)^{\frac{1}{1+2\alpha+\beta}}, \quad \psi = \left(\frac{1+2\alpha+\beta}{1+\alpha+\beta}\right)^{\frac{1+\alpha+\beta}{1+2\alpha+\beta}}\Lambda\left(\frac{x}{\Lambda}\right)^{\frac{1+\alpha+\beta}{1+2\alpha+\beta}} \tag{3.5}$$

Because of the above, the solutions (3.5) possess different accuracy for a given x. For instance, for $\Lambda = 10^{-3}$, $\alpha = 1$, $\beta = 0.5$, $x = 10^2$ the errors in evaluating g and ψ are 12 and 32%, respectively. As x is doubled, they diminish to 7 and 19%. It is interesting to note also the following property of the solutions for $\Lambda \neq 0$. In conformity with (3.5), for $x > \Lambda^{-(1+\alpha)/(\alpha+\beta)}$ the thermal layer thickness ψ becomes larger than the thermal constant g while for $\Lambda = 0$ always $\psi < g$. However, it should be recalled that for very large g the solution of the problem is formal in nature if we go outside the framework of the assumptions made in the initial physical model.

4. The solution in the intermediate domain is obtained by numerical integration of (2.7). The integral curves for the case $\alpha = 1$, $\beta = 0.5$ (air under normal conditions) are presented in Figs. 2 and 3 (numbers at the curves are values of Λ). As the heat elimination grows the layer thickness increases, and the maximal value of the enthalpy (thinning out) drops on the layer axis. It is seen that for small x the curves merge in conformity with (3.1). The dashed curves $\Lambda = 10^{-3}$ are evaluated by the asymptotic formulas (3.5) and illustrate the accuracy of the expansions mentioned. For $\Lambda = 10^{-5}$ the error in evaluating ψ varies between 40 and 60% for $200 < x < 500$ according to (3.5). A calculation by (3.4) with $x_2 = 100$ will yield an accuracy on the level of several percent (the dashed line $\Lambda = 10^{-5}$ in Fig. 3). The dashed line $\Lambda = 10^{-5}$ in Fig. 2 is also evaluated for $x_1 = 200$ according to (3.4) (evaluation of g by (3.5) yields the same result within 10% limits).

In conclusion, it should be noted that the integral method used in this paper will visibly yield values of the magnitudes of the enthalpy and density on the axis most exactly, which are of interest in practice.

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METHOD OF MULTIEXPOSURE PHOTOGRAPHIC RECORDING OF PARTICLES IN HIGH-VELOCITY TWO-PHASE FLOWS

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There is a broad circle of experimental problems in the gasdynamics of multiphase systems when it is important to assure a high fast response of the measuring circuit in addition to the necessity to measure high velocities of 10-10^4 m/sec. Here are problems that occur in studying two-phase pulsed flows, particle dynamics behind shocks in investigations of heterogeneous detonation or deposition of detonation coatings, etc., when the characteristic times of the processes are $\sim 10^{-1}$-10^{-5} sec.

It should be noted that velocity measurement in the range 10-10^4 m/sec assures utilization of laser-Doppler systems with direct spectrum analysis examined in detail in the survey paper [1]. However, the practical realization of laser doppler velocimeter (LDV) circuits with a resolution time of $\sim 10^{-5}$-10^{-6} sec requires the development of special methods of recording the spectra, which constrains their extension to the area of problems related to the investigation of high-speed processes.

Here the development of a method of multiexposure photographic recording, based on the use of a stroboscopic light source yielding the frequency and duration of frame exposure, is of significant interest. By applying such a source in combination with different optical schemes (shadow, interferometer, holographic), extensive information can be obtained about the flow structure and particle parameters such as size, concentration, and velocity of their motion, can be determined.

Up to now, a number of papers [2-6] is known in which multiexposure photorecording was used to solve different problems, for instance, to investigate turbulent flows [2-3], measure drop velocity [4], convective fluid motion [5], particle free fall [6], etc.; however, all these papers refer to low velocity measurements $\lesssim 10$ m/sec.

The development of powerful pulse lasers as well as optical systems and photographic materials with high resolution permits significant expansion of the possibilities of this method by increasing its response, and spatial and time resolution. Utilization of a spatial-spectral method of analyzing multiexposure photographs [6] affords the possibility of substantially simplifying data processing and executing measurements in a broad range of concentrations. This paper is devoted to development of a multiexposure photorecording method to investigate rapidly progressing processes in heterogeneous flows.

1. In principle, the scheme for multiexposure recording of a particle image is the following. Several successive focussed images of a moving two-phase flow are recorded at equal time intervals Δt on the very

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